Thermal Stress and Instability of Sandwich Cylinders on Rigid Supports

R. A. Gellatly*, P. P. Bijlaard†, and R. H. Gallagher‡
Bell Aerosystems Company, Buffalo, N. Y.

Solutions are presented for the thermal stresses in uniformly heated sandwich cylindrical shells for conditions of both simple and fixed support. These cases refer to a single-bay cylinder and a cylinder with many bays, respectively, where the bulkheads or rings are rigid in their planes, but provide no restraint against rotation. Solutions for the temperature change to cause buckling are also presented. Numerical results in the form of design charts are given for practical ranges of the pertinent parameters for both thermal stresses and the critical temperatures for buckling.

Nomenclature

a	=	mean radius of cylinder, in.
C_p	=	coefficient of pth harmonic in Fourier series representation of hoop stress
c	=	cylinder half-length, in.
D_c	=	stiffness parameter
D_Q	=	sandwich shear stiffness, lb/in.
D_s	=	sandwich bending stiffness, in.-lb
E	=	modulus of elasticity, psi
G_c	=	core shear modulus, psi
H_d, H_l	=	nondimensional parameters
h	=	core depth, in.
M	=	bending moment per unit width, in.-lb/in.
Q_x	=	transverse shear force, lb/in.
ΔT	=	temperature change from stress-free state, °F
t	=	face thickness, in.
t'	=	isotropic plate thickness, in.
w	=	radial displacement, in.
α	=	coefficient of thermal expansion, in./in.-°F
λ	=	cylinder length-to-radius ratio
Π	=	buckling stress parameter
σ_y	=	hoop stress, psi

1. Introduction

As a result of advances in fabricational technology, sandwich construction is receiving continuously greater consideration for use as the primary structure for cylindrical portions of missile, aircraft, and space vehicles. Also, because of increasing performance requirements, certain classes of each of these types of vehicles can be expected to sustain significant aerodynamic heating. An important structural analysis problem that arises under such circumstances concerns the prediction of the thermal stresses produced by the heating of a sandwich cylinder, which is supported by extremely stiff rings or bulkheads. This problem is treated in the present paper. Also dealt with is the prediction of buckling under the subject conditions

The conditions of analysis are illustrated in Fig. 1. The cylinder is of mean radius a and core thickness h with isotropic faces of equal thickness t. The faces possess only in-plane stiffness and have a modulus of elasticity E, whereas the core possesses only a transverse shear stiffness as defined by the modulus of rigidity G_c. The cylinder is of finite length $2c$ and is supported at each end by a rigid bulkhead. The coordinate system is located at the center of the cylinder for purposes of developing the thermal stress solution, but it is shifted to the end of the cylinder for convenience in formulating the buckling problem. The cylinder is uniformly heated so that a temperature change ΔT from the stress-free state is sustained; the bulkheads are unheated.

Two support conditions are examined: simple and fixed support. The simple support condition may serve as an approximation to the case of cylinders, such as pressure vessels, which are supported by closures at the ends but which have no axial bending moment continuity with adjacent structures (Fig. 1). The fixed-support condition is useful for representation of long, ring-stiffened, cylinders (Fig. 2).

2. Thermal Stresses

The subject problem is similar to that of a beam on an elastic foundation. Using this concept, the basic differential equation for the radial deflection (w) is

$$\frac{d^4w}{dx^4} - \frac{2tE}{a^2D_Q}\frac{d^2w}{dx^2} + \frac{2tE}{a^2D_s}(w + a\alpha\Delta T) = 0 \qquad (1)$$

where α is the coefficient of linear thermal expansion. D_Q and D_s are the sandwich shear and flexural stiffnesses, respectively. Formulas for these stiffnesses, in terms of h, G_c, t, and E are recommended in Ref. 1. Simple, appropriate, formulas for these stiffnesses are

$$D_Q = (h + t)G_c \qquad D_s = Et(h + t)^2/2(1 - \mu^2)$$

Because of symmetry, the general solution for Eq. (1) reduces to

$$w = C_1 \cosh m_1\left(\frac{Et}{a^2D_Q}\right)^{1/2} x + C_2 \cosh m_2\left(\frac{Et}{a^2D_Q}\right)^{1/2} x - a\alpha\Delta T \qquad (2)$$

where m_1 and m_2 follow from the auxiliary equation (in nondimensional form) as

$$m_1^2 = \{1 + [1 - (2/H_d)]^{1/2}\}$$
$$m_2^2 = \{1 - [1 - (2/H_d)]^{1/2}\} \qquad (3)$$

with $H_d = EtD_s/a^2D_Q^2$.

The roots m_1 and m_2 may be real ($H_d > 2$) or complex ($H_d < 2$). In the latter case, the hyperbolic functions of Eq. (2) should be modified as follows:

$$\cosh(x + iy) = (\cosh x)(\cos y) + i(\sinh x)(\sin y)$$

Received May 7, 1964; revision received September 25, 1964. Work described herein was performed in conjunction with a Federal Aviation Agency sponsored study, Contract AF33(657)-8936, "Thermal Stress Determination Techniques in Supersonic Transport Aircraft Structures."

* Structures Research Engineer, Aerospace Engineering Department.

† Consultant; also Professor of Engineering Mechanics, Cornell University, Ithaca, New York. Member AIAA.

‡ Chief, Advanced Airframe Analysis. Associate Fellow Member AIAA.

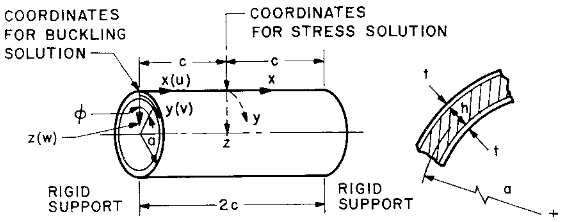

Fig. 1 Conditions of analysis.

If all computations are carried out in a complex mode, it is not necessary to transform the hyperbolic functions.

The constants C_1 and C_2 are determined from the support conditions. Consider first the simple support case where at $x = \pm c$:

$$w = 0 \qquad M_x = -D_s[(d^2w/dx^2) - (2Et/a^2D_Q)(w + a\alpha\Delta T)] = 0$$

Using these conditions, the constants C_1 and C_2 are determined and the deflection equation becomes

$$w = \frac{a\alpha\Delta T}{(m_2{}^2 - m_1{}^2)}\left[(m_2{}^2 - 2)\frac{\cosh m_1H_x}{\cosh m_1H_l} - (m_1{}^2 - 2)\frac{\cosh m_2H_x}{\cosh m_2H_l} - (m_2{}^2 - m_1{}^2)\right] \quad (4)$$

where

$$H_x = (Et/D_Q)^{1/2}(x/a) \qquad H_l = (Et/D_Q)^{1/2}(c/a) \quad (4a)$$

With built-in ends, the boundary conditions are, at $x = \pm c$,

$$w = 0 \qquad dw/dx = Q_x/D_Q$$

where Q_x is the transverse shear force and is given by

$$Q_x = -D_s\left[\frac{d^3w}{dx^3} - \frac{2t}{a^2}\frac{E}{D_Q}\frac{dw}{dx}\right] \quad (5)$$

so that the second boundary condition becomes

$$\frac{dw}{dx}(1 - 2H_d) + \frac{D_s}{D_Q}\frac{d^3w}{dx^3} = 0 \quad (6)$$

Use of the foregoing boundary conditions with Eq. (2) and its derivatives yields the constants C_1 and C_2 for built-in support conditions and leads to the following expression for the radial displacement:

$$w = a\alpha\Delta T\left[\frac{[1 - H_d(2 - m_2{}^2)]\sinh m_2H_l\cosh m_1H_x - [1 - H_d(2 - m_1{}^2)](m_1/m_2)\sinh m_1H_l\cosh m_2H_x}{[1 - H_d(2 - m_2{}^2)]\sinh m_2H_l\cosh m_1H_l - [1 - H_d(2 - m_1{}^2)](m_1/m_2)\sinh m_1H_l\cosh m_2H_l} - 1\right] \quad (7)$$

The foregoing solutions for radial displacement can be transformed into equations for hoop stress (σ_y) and longitudinal bending moment (M_x) by use of the following basic relationships:

$$\sigma_y = -(E/a)[w + a\alpha\Delta T] \quad (8)$$

$$M_x = -D_s[(d^2w/dx^2) - (2tE/a^2D_Q)(w + a\alpha\Delta T)] \quad (9)$$

Equations for the hoop stress and bending moment at the center of the cylinder and bending moment at the support are given in Table 1. These were used to evaluate the hoop stresses and bending moments for both support conditions for realistic ranges of the pertinent parameters (H_l and H_d). These results are plotted in the form of nondimensionalized design charts in Figs. 3–7. These figures show the characteristic reversal of stresses associated with the beam-on-elastic-foundation type of problem. An examination of the accuracy of the plotted data and the presentation of illustrative examples follows the development of the instability solution.

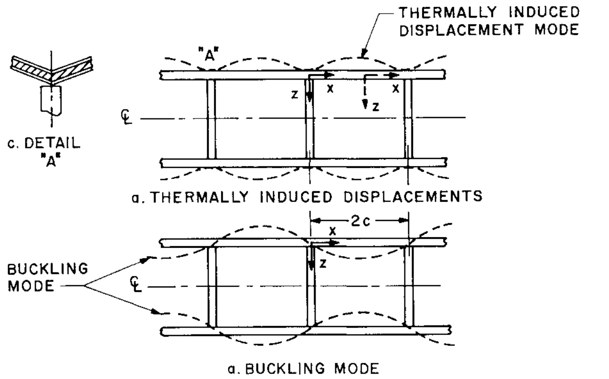

Fig. 2 Multibay cylinder: thermally induced displacements and buckling mode.

3. Thermal Buckling

From Eq. (8) and from the corresponding formulations of Table 1, it is seen that the hoop thermal stress equation for both support conditions can be written in the general form (considering compressive stress as positive)

$$\sigma_y = E\alpha\Delta T(A\cosh m_1H_x + B\cosh m_2H_x) \quad (10)$$

where m_1, m_2, and H_x follow from Eqs. (3) and (4a). Shifting the origin of the coordinate system from the center of the cylinder to the left end of a bay (Fig. 1) Eq. (10) transforms to

$$\sigma_y = E\alpha\Delta T[A\cosh m_1(H_x - H_l) + B\cosh m_2(H_x - H_l)] \quad (11)$$

which can be expressed by the following Fourier series:

$$\sigma_y = E\alpha\Delta T\left[\frac{C_0}{2} + \sum_{r=1}^{\infty} C_r\cos\frac{r\pi x}{2c}\right] \quad (12)$$

where

$$C_r = \frac{2Am_1H_l\sinh m_1H_l}{(r\pi/2)^2 + (m_1H_l)^2} + \frac{2Bm_2H_l\sinh m_2H_l}{(r\pi/2)^2 + (m_2H_l)^2} \quad (13)$$

for r even, and $C_r = 0$ for r odd.

Buckling is governed by Donnell's equation as modified to apply to sandwich construction[3]:

$$D_s\nabla^8w + 2t\left(1 - \frac{D_s}{D_Q}\nabla^2\right)\left(\frac{E}{a^2}\frac{\partial^4w}{\partial x^4} + \nabla^4\frac{\sigma_y}{a^2}\frac{\partial^2w}{\partial\phi^2}\right) = 0 \quad (14)$$

Using nondimensional coordinates where $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial\phi^2)$ and x stands for x/a, this becomes

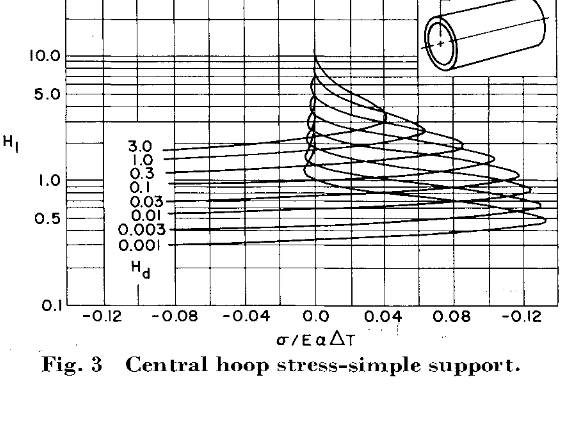

Fig. 3 Central hoop stress-simple support.

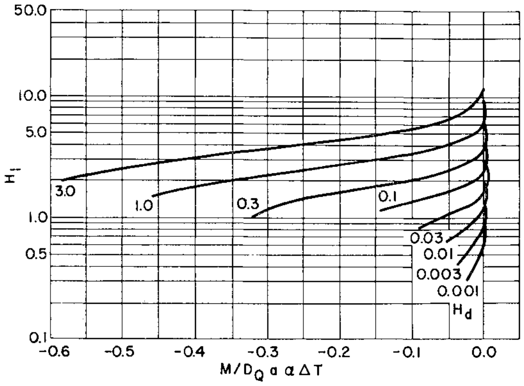

Fig. 4 Central bending moment-simple support.

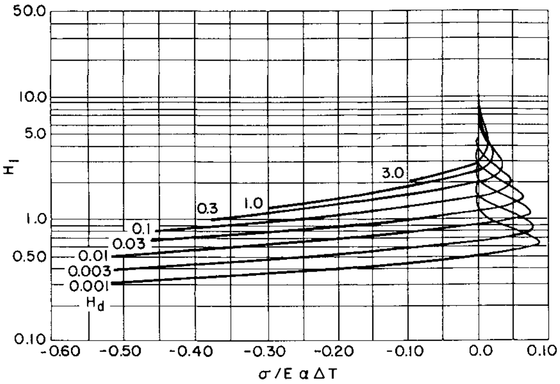

Fig. 5 Central hoop stress-fixed supports.

$$\nabla^8 w + 4(1-\mu^2)\left(\frac{a}{h+t}\right)^2\left(1 - \frac{D_s}{D_Q a^2}\nabla^2\right)\frac{\partial^4 w}{\partial x^4} + 4(1-\mu^2)\left(\frac{a}{h+t}\right)^2\left(1 - \frac{D_s}{D_Q a^2}\nabla^2\right)\nabla^4 \frac{\sigma_y}{E}\frac{\partial^2 w}{\partial \phi^2} = 0 \quad (15)$$

In case of a single cylinder as well as for a multibay cylinder with rings (Fig. 2b), buckling will occur in the simple-supported mode, so that one can assume

$$w = \cos n\phi \Sigma b_r \sin(r\pi x/\lambda) \quad (16)$$

where

$$\lambda = 2c/a \quad (17)$$

Then Eq. (15) may be written as

$$\nabla^8 w + 4(1-\mu^2)\left(\frac{a}{h+t}\right)^2\left\{1 + \frac{D_s}{D_Q a^2}\left[\left(\frac{r\pi}{\lambda}\right)^2 + n^2\right]\right\} \times \frac{\partial^4 w}{\partial x^4} + 4(1-\mu^2)\left(\frac{a}{h+t}\right)^2 \times \left\{1 + \frac{D_s}{D_Q a^2}\left[\left(\frac{r\pi}{\lambda}\right)^2 + n^2\right]\right\}\nabla^4 \frac{\sigma_y}{E}\frac{\partial^2 w}{\partial \phi^2} = 0 \quad (18)$$

Comparison of this differential equation with that for a homogeneous shell, as used in Ref. 2 in Eq. (1), namely,

$$\nabla^8 w + 4K^4 \frac{\partial^4 w}{\partial x^4} + 4K^4 \nabla^4 \frac{\sigma_y}{E}\frac{\partial^2 w}{\partial \phi^2} = 0 \quad (19)$$

shows that it also applies to the sandwich shell if the constant $4K^4$ is given by

$$4K^4 = 4(1-\mu^2)\left(\frac{a}{h+t}\right)^2\left\{1 + \frac{D_s}{D_Q a^2}\left[\left(\frac{r\pi}{\lambda}\right)^2 + n^2\right]\right\} \quad (20)$$

Hence, this analysis can follow that of Hoff,[2] suitably modified to include shear deformation terms. Substitution of Eqs. (12) and (16) into Eq. (19) then leads to an equation similar to Eq. (27) of Ref. 2:

$$\sum_{q=1}^{q=r-1} b_q(C_{r-q} - C_{r+q}) - b_r\left(\frac{U_r}{\alpha \Delta T} - C_0 + C_{2r}\right) + \sum_{q=r+1}^{\infty} b_q(C_{q-r} - C_{q+r}) = 0 \quad (21)$$

where

$$U_r = \frac{[(r\pi/\lambda)^2 + n^2]^4 - 4K^4(r\pi/\lambda)^4}{2K^4 n^2[(r\pi/\lambda)^2 + n^2]^2} \quad (22)$$

where now K^4 follows from Eq. (20). Equation (21) can now be written for $r = 1, 2, 3 \ldots p$, which yields p simultaneous equations. The condition of a vanishing determinant leads to the critical value of the buckling stress parameter $\Pi = \alpha \Delta T_{cr}$. It is, however, more convenient, as was done in Ref. 4, to present this problem in matrix form and apply an iterative technique for the determination of Π

Table 1 Formulas for hoop stress and longitudinal bending moment

A. Simple support

Hoop stress at center (midway between supports)

$$\sigma_y = \frac{E\alpha\Delta T}{(m_1^2 - m_2^2)}\left[\frac{(m_2^2 - 2)}{\cosh m_1 H_l} - \frac{(m_1^2 - 2)}{\cosh m_2 H_l}\right]$$

Longitudinal bending moment at center

$$M_x = H_d D_Q a \alpha \Delta T \frac{(m_1^2 - 2)(m_2^2 - 2)}{(m_1^2 - m_2^2)}\left[\frac{1}{\cosh m_1 H_l} - \frac{1}{\cosh m_2 H_l}\right]$$

B. Fixed support

Hoop stress at center

$$\sigma_y = -E\alpha\Delta T\left[\frac{[1 - H_d(2 - m_2^2)]\sinh m_2 H_l - [1 - H_d(2 - m_1^2)](m_1/m_2)\sinh m_1 H_l}{[1 - H_d(2 - m_2^2)]\sinh m_2 H_l \cosh m_1 H_l - (m_1/m_2)[1 - H_d(2 - m_1^2)]\sinh m_1 H_l \cosh m_2 H_l}\right]$$

Longitudinal bending moment at center

$$M_x = -H_d D_Q a \alpha \Delta T\left[\frac{[1 - H_d(2 - m_2^2)](m_1^2 - 2)\sinh m_2 H_l - [1 - H_d(2 - m_1^2)](m_2^2 - 2)(m_1/m_2)\sinh m_1 H_l}{[1 - H_d(2 - m_2^2)]\sinh m_2 H_l \cosh m_1 H_l - [1 - H_d(2 - m_1^2)](m_1/m_2)\sinh m_1 H_l \cosh m_2 H_l}\right]$$

Longitudinal bending moment at support

$$M_x = -H_d D_Q a \alpha \Delta T\left[\frac{[1 - H_d(2 - m_2^2)](m_1^2 - 2)\sinh m_2 H_l \cosh m_1 H_l - [1 - H_d(2 - m_1^2)](m_2^2 - 2)(m_1/m_2)\sinh m_1 H_l \cosh m_2 H_l}{[1 - H_d(2 - m_2^2)]\sinh m_2 H_l \cosh m_1 H_l - [1 - H_d(2 - m_1^2)](m_1/m_2)\sinh m_1 H_l \cosh m_2 H_l}\right]$$

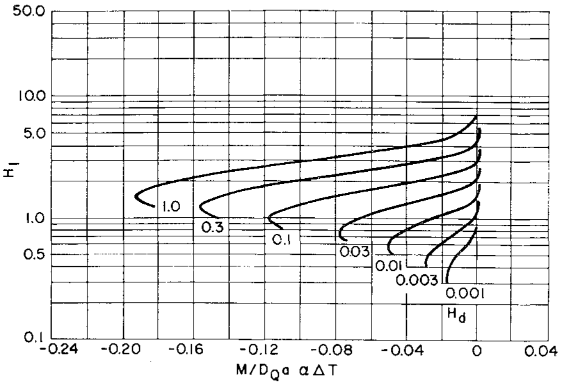

Fig. 6 Central bending moment-fixed supports.

for a number of values of the wave length parameter n. When the simultaneous equations are written in matrix form, one obtains

$$[U]\{b\} + \Pi[\Sigma]\{b\} = 0 \tag{23}$$

or

$$\frac{1}{\Pi}\{b\} = -[U]^{-1}[\Sigma]\{b\} \tag{24}$$

In the present case, the matrix $[U]$ is a diagonal matrix and is therefore easily inverted. Using matrix iterative techniques, the eigenvalues of the system for a range of values were computed and the minimum determined using a standard curve fitting procedure. Details are presented in Ref. 5. The eigenvalues $\Pi = \alpha\Delta T_{cr}$ give the critical temperature rise ΔT_{cr}. The results are presented graphically in Figs. 8 and 9, for simply supported single-bay cylinders and for many-bay cylinders, respectively. In both cases, buckling occurs in a simply supported mode (Figs. 2a and 2b), but in the first case the thermal stresses are those for the simply supported case whereas in the second case they are for the clamped case.

The pertinent parameters in Figs. 8 and 9 are in the same range of values as were employed for the design curves in Figs. 3–7. Both sets of curves have been terminated at an upper value $Et\alpha\Delta T_{cr}/D_Q = 0.5$, corresponding to the shear instability failure of the core. This instability is characterized by the relationship

$$2t\sigma_y = D_Q \tag{25}$$

A very simple derivation of this relation is given in Ref. 6. At the restrained ends of the bay, the hoop stress σ_y equals $E\alpha\Delta T_{cr}$. Thus, $2tE\alpha\Delta T_{cr}/D_Q = 1.0$ or $Et\alpha\Delta T_{cr}/D_Q = 0.5$. The significance of the present data with respect to published isotropic cylinder solutions is discussed at length in the third of the four following illustrative examples.

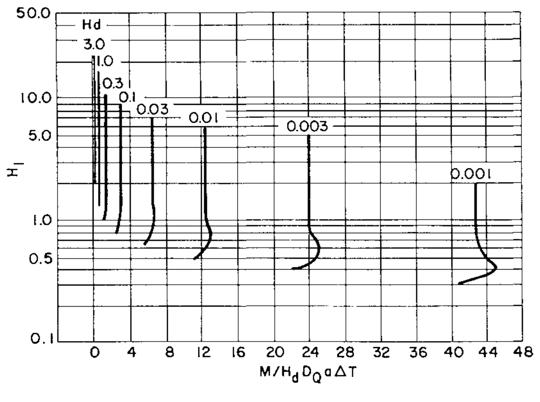

Fig. 7 End moment-fixed supports.

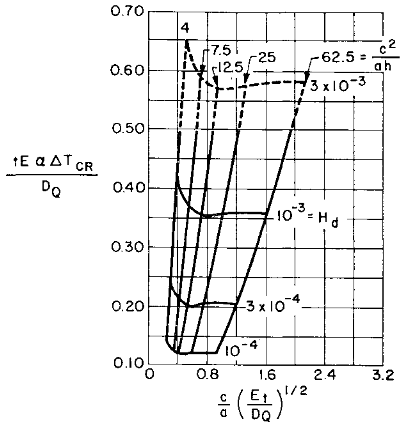

Fig. 8 Critical temperature: simple support, single bay.

4. Illustrative Examples

A. Isotropic Cylinder: Thermal Stress Analysis

As a check on the accuracy of the plotted data, the case of an isotropic cylinder with simply supported ends examined by Hoff[2] will be treated. (No alternative solutions were found for the sandwich problem.) In Hoff's paper, a = 10 in., c = 1.57 in., $E = 29 \times 10^6$ psi, $\mu = 0.3$, and the wall thickness t' is 0.0331 in.

The bending stiffness of the cylinder is

$$D_s = E(t')^3/12(1 - \mu^2) = 96.307 \text{ in.-lb}$$

A core shear stiffness D_Q must be defined. For isotropic plates the shear deformation is ordinarily neglected ($G_c = \infty$) and a large value for D_Q may be chosen arbitrarily. Thus, assume $D_Q = 2.1 \times 10^4$ psi. Also, the "equivalent" sandwich face thickness (t) must be $t'/2$ since the total cross-sectional areas must be the same in each case. Hence,

$$H_d = EtD_s/a^2D_Q^2 = 10^{-3}$$

$$H_l = (c/a)(Et/D_Q)^{1/2} = 0.742$$

From Fig. 3 at $H_d = 10^{-3}$ and $H_l = 0.742$, the hoop stress at center

$$\sigma_y = 0.057E\alpha\Delta T$$

In Ref. 2, the stress at the center of the cylinder is obtained from the expression

$$\sigma_y = -\left[\frac{\cosh\beta\cos\beta}{\sinh^2\beta + \cos^2\beta}\right]E\alpha\Delta T$$

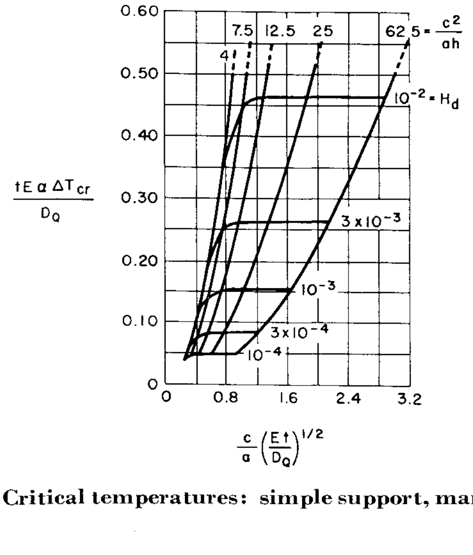

Fig. 9 Critical temperatures: simple support, many bays.

where $\beta = 0.643(4c^2/at')^{1/2} = 3.51$. The stress, as derived from the foregoing formula, is then

$$\sigma_y = 0.056E\alpha\Delta T$$

Thus, there is excellent agreement between the graphical and analytical results.

B. Sandwich Cylinder: Thermal Stress Analysis

As an indication of the thermal stresses produced in a sandwich cylinder under practical conditions and to illustrate further the use of the design curves, consider the case of a long sandwich cylinder of 70-in. mean radius, ring stiffened at intervals of 48 in. The core thickness is 1.0 in., the face thicknesses 0.030 in. Both the core and the faces are composed of 17-7PH stainless steel, $E = 27 \times 10^6$ psi, $\mu = 0.3$, $\alpha = 6.1 \times 10^{-6}/°F$, $G_c = 100{,}000$ psi. The cylinder is uniformly heated to a temperature 600°F in excess of the ring temperature. Assuming the ring stiffeners are infinitely rigid and that in any particular bay the cylinder can be assumed to be fixed-supported at its ends, determine the resulting hoop stresses and longitudinal bending moments. For this case,

$$D_Q = (h + t)G_c = 103{,}000 \text{ psi}$$

$$D_s = \frac{Et(h+t)^2}{2(1-\mu^2)} = 4.7215 \times 10^5 \text{ in.-lb}$$

Thus

$$H_d = \frac{EtD_s}{a^2D_Q^2} = 7.35 \times 10^{-3} \qquad H_l = \left[\frac{Etc}{a^2D_Q}\right]^{1/2} = 0.961$$

From Fig. 5 by interpolation for H_d and H_l, the hoop stress at the center is given by

$$\sigma_y = 0.060E\alpha\Delta T = 5930 \text{ psi}$$

From Figs. 6 and 7, the bending moments at the center and ends are found to be as follows:

At Center

$$M_{xc} = -0.0152D_Qa\alpha\Delta T = -401 \text{ in.-lb/in.}$$

At Ends

$$M_{xe} = 0.12D_Qa\alpha\Delta T = 3165 \text{ in.-lb/in.}$$

It may be mentioned that in multibay cylinders, because $dw/dx \neq 0$ at the rings (Fig. 2c), there will be, moreover, large bending stresses in the faces from bending about their individual middle planes.

C. Isotropic Cylinder: Instability Analysis

A 17-7 PH stainless-steel isotropic cylinder of radius 15 in., wall thickness 0.022 in., is simply supported at rigid bulkheads at intervals of 5 in. The critical temperature is to be determined for both the single-bay and multibay support conditions. As before, $t = t'/2 = 0.011$ in.:

$$D_s = E(t')^3/12(1 - \mu^2) = 26.32 \text{ in.-lb}$$

The shear stiffness of the core is given an arbitrary large value $D_Q = 1.86 \times 10^4$. Thus,

$$H_d = EtD_s/a^2D_Q^2 = 10^{-4}$$

$$H_l = [Etc^2/a^2D_Q]^{1/2} = 0.665$$

From Figs. 8 and 9 for a single-bay cylinder

$$Et\alpha\Delta T_{cr}/D_Q = 0.121 \quad \text{or} \quad \Delta T_{cr} = 1245 \text{ °F}$$

and for a multibay cylinder

$$Et\alpha\Delta T_{cr}/D_Q = 0.049 \quad \text{or} \quad \Delta T_{cr} = 504 \text{ °F}$$

In order to provide a direct comparison with the solutions given by Anderson,[7] the foregoing results must be related to the parameter $(\alpha\Delta T_{cr})(a/t')$ and to the quantity $(2c)^2/at'$. For the current cylinder $(2c)^2/at' = 75.0$. Thus, for a single-bay cylinder

$$(\alpha\Delta T_{cr})\left(\frac{a}{t'}\right) = \left(0.121\frac{D_Q}{Et}\right)\left(\frac{a}{t'}\right) = 5.18$$

and for a multibay cylinder

$$(\alpha\Delta T_{cr})\left(\frac{a}{t'}\right) = \left(0.049\frac{D_Q}{Et}\right)\left(\frac{a}{t'}\right) = 2.09$$

Corresponding values of these parameters, as read from Fig. 1 of Ref. 7, for a cylinder with $(2c)^2/at' = 75.0$, are 5.24 and 2.08, respectively.

D. Sandwich Cylinder: Instability Analysis

A multibay sandwich cylinder of mean radius 95 in. is simply supported over rigid rings at intervals of 48 in. The core thickness is 1.0 in. and the face thicknesses 0.030 in. The faces and core are composed of 17-7 PH stainless steel, but now the shear rigidity of the core is $G_c = 63{,}200$ psi. Determine the critical temperature rise. Thus,

$$D_Q = (h + t)G_c = 65{,}100 \text{ lb/in.}$$

$$D_s = Et(h + t)^2/2(1 - \mu^2) = 4.7215 \times 10^5 \text{ in.-lb}$$

$$H_d = EtD_s/a^2D_Q^2 = 10^{-2}$$

$$H_l = [Etc^2/a^2D_Q]^{1/2} = 0.890$$

From Fig. 9, the solution for these values of H_d and H_l yields

$$Et\alpha\Delta T_{cr}/D_Q = 0.4$$

so that $\Delta T_{cr} = 5330$°F.

It can be seen from the foregoing examples that the subject-type of instability is unlikely to occur in sandwich cylinders of normal proportions. It is, however, a possible mode of failure in thin-walled cylinders of large radius.

In closing, it should be noted that, when a sandwich cylinder is subjected to aerodynamic heating, it is likely to sustain significant thermal stresses due to a difference between the temperatures of the faces. Design curves for this problem are given in Ref. 8. To account for flexible rather than rigid supports requires the introduction of additional parameters, which limits the likelihood of deriving design curves. One can consult Ref. 9 in order to ascertain procedures for accounting for flexible supports.

References

[1] "Sandwich construction for aircraft," Air Force, Navy, Commerce Bull. ANC-23, Part II (1955).

[2] Hoff, N. J., "Buckling of thin cylindrical shell under hoop stresses varying in the axial direction," J. Appl. Mech. **24,** 405–412(1957).

[3] Stein, M. and Mayers, J., "A small-deflection theory for curved sandwich plates," NACA TN 2017 (February 1950).

[4] Bijlaard, P. P. and Gallagher, R. H., "Elastic instability of a cylindrical shell under arbitrary circumferential variation of axial stress," J. Aerospace Sci. **27,** 854–858 (1960).

[5] Gellatly, R. A. and Gallagher, R. H., "Thermal stress determination techniques for supersonic transport aircraft structures, Part II, Design data for sandwich plates and cylinders under applied loads and thermal gradients," Aeronautical Systems Div. TDR 53-783, Part II (January 1964).

[6] Bijlaard, P. P., Discussion of paper "Buckling of sandwich cylinders under combined compression, torsion, and bending loads," J. Appl. Mech. **23,** 157–158(1956).

[7] Anderson, M. S., "Thermal buckling of cylinders," *Collected Papers on Instability of Shell Structures* NASA TN D-1510 (December 1962).

[8] Gellatly, R. A. and Gallagher, R. H., "Stresses in sandwich cylinders," Machine Design **36,** 197–201(March 26, 1964).

[9] Bijlaard, P. P. "Buckling under external pressure of cylindrical shells evenly stiffened by rings only," J. Aeronaut. Sci. **24,** 437–447(1957).